

$U(2)_L \times U(2)_R$ Chiral Theory of Mesons

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Abstract

A $U(2)_L \times U(2)_R$ chiral theory of pseudoscalar, vector, and axial-vector mesons has been proposed. VMD has been revealed from this theory. The physical processes of normal parity and abnormal parity have been studied by using the same lagrangian and the universality of coupling has been revealed. Two new mass relations between vector and axial-vector mesons have been found. Weinberg's first sum rule and new relations about the amplitude of a_1 decay are satisfied. KSFR sum rule is satisfied pretty well. The ρ pole in pion form factor has been achieved. The theoretical results of $\rho \rightarrow \pi\pi$, $\omega \rightarrow \pi\pi$, $a_1 \rightarrow \rho\pi$ and $\pi\gamma$, $\tau \rightarrow \rho\nu$, $\tau \rightarrow a_1\nu$, $\pi^0 \rightarrow \gamma\gamma$, $\omega \rightarrow \pi\gamma$, $\rho \rightarrow \pi\gamma$,

$f_1 \rightarrow \rho\pi\pi$, $f_1 \rightarrow \eta\pi\pi$, $\rho \rightarrow \eta\gamma$, $\omega \rightarrow \eta\gamma$ are in good agreement with data. Weinberg's $\pi\pi$ scattering lengths and slopes and a_2^0 , a_2^2 , and b_1^1 have been obtained. Especially, the ρ resonance in the amplitude T_1^1 of $\pi\pi$ scattering has been revealed from this theory. Two coefficients of chiral perturbation theory have been determined and they are close to the values used by chiral perturbation theory.

Chiral symmetry is one of the most important features revealed from quantum chromodynamics (QCD). The chiral perturbation theory(CPT) is successful in describing the pseudoscalar meson physics[1,2]. Chiral lagrangian has been used to study physics of vector and axial-vector mesons before QCD [3]. Weinberg's sum rules[4] of ρ and a_1 mesons, KSRF sum rule[5] of ρ meson are also among earlier works in this field. On the other hand, in ref.[6] in terms of large N_c expansion t'Hooft argues that QCD is equivalent to a meson theory at low energies. In principle, all mesons should be included in this meson theory. Of course, in chiral perturbation theory the effects of other mesons have been included in the coefficients of the lagrangian up to $O(p^4)$. As a matter of fact, in ref.[7,8] the authors have found that the vector meson dominates the structure of the phenomenological chiral lagrangian. Various effective theory including ρ and a_1 mesons has been studied in last decade[9]. Wess-Zumino lagrangian[10] is an important part of the effective meson theory. Witten[11] and other authors[12,13] have generalized Wess-Zumino lagrangian to include vector and axial-vector mesons by the requiring gauge invariance. In refs.[12,14] the generalized Wess-Zumino terms have been used to study meson physics. In this paper a $U(2)_L \times U(2)_R$ chiral theory of mesons including pseudoscalar, vector and axial-vector has been studied. The paper is organized as follows. 1) the formalism of the theory; 2) the definitions of the physical fields; 3) new mass relations between ρ a_1 and ω $f_1(1285)$; 4) VDM; 5) the decays of $\rho \rightarrow 2\pi$ and $\omega \rightarrow 2\pi$; 6) pion form factor; 7) the decays of $a_1 \rightarrow \rho\pi$ and $a_1 \rightarrow \gamma\pi$; 8) revisit of Weinberg's

sum rules; 9) decays of $\tau \rightarrow \rho\nu$ and $\tau \rightarrow a_1\nu$; 9) $\pi\pi$ scattering and determination of the coefficients of CPT; 10) decays of $\omega \rightarrow \rho\pi$, $\omega \rightarrow \gamma\pi$, $\rho \rightarrow \gamma\pi$, and $\pi^0 \rightarrow \gamma\gamma$; 11) decays of $f_1(1285)$; 12) decays of $\rho \rightarrow \eta\gamma$ and $\omega \rightarrow \eta\gamma$; 13) Large N_c expansion; 14) Dynamical chiral symmetry breaking; 15) Derivative expansion; 16) Summary of the results.

The formalism of $U(2)_L \times U(2)_R$ chiral theory of mesons

In this paper only two flavors are taken into account and we do not need to worry about the processes forbidden by OZI rule. η' meson will not be discussed. Therefore, $U(1)$ problem is not an issue of this paper. In flavor space the mesons are coupled to quarks only. The background field method is a convenient way of deriving effective lagrangian of mesons. The ingredients of this effective meson theory are pseudoscalar mesons (pions and u and d quark components of η), vector mesons (ρ and ω), axial-vector mesons (a_1 and $f_1(1285)$), quarks, leptons, photon, and W bosons. Using $U(2)_L \times U(2)_R$ chiral symmetry and the minimum coupling principle, the lagrangian has been constructed as

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(x)(i\not{\partial} + \not{p} + e_0 Q \not{A} + \not{p}\gamma_5 - mu(x))\psi(x) + \frac{1}{2}m_0^2(\rho_i^\mu \rho_{\mu i} + \omega^\mu \omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu) \\ & + \bar{\psi}(x)_L \not{W} \psi(x)_L + \mathcal{L}_{EM} + \mathcal{L}_W + \mathcal{L}_{lepton} \end{aligned} \quad (1)$$

where $a_\mu = \tau_i a_\mu^i + f_\mu$, $v_\mu = \tau_i \rho_\mu^i + \omega_\mu$, A_μ is photon field, $Q = \frac{\tau_3}{2} + \frac{1}{6}$ is the electric charge operator of u and d quarks, W_μ^i is W boson, and $u = \exp\{\gamma_5(\tau_i \pi_i + \eta)\}$, m is a parameter.

In eq.(1) u can be written as

$$u = \frac{1}{2}(1 + \gamma_5)U + \frac{1}{2}(1 - \gamma_5)U^\dagger, \quad (2)$$

where $U = \exp(i(\tau_i \pi_i + \eta))$. Mesons are bound states solutions of QCD and in QCD mesons are not independent degrees of freedom. Therefore, in eq.(1) there are no kinetic terms for meson fields. The kinetic terms of meson fields are generated from quark loops. Using method of path integral to integrating out the quark fields, the effective lagrangian of mesons(indicated by M) are obtained

$$\exp\{i \int d^4x \mathcal{L}^M\} = \int [d\psi][d\bar{\psi}] \exp\{i \int d^4x \mathcal{L}\}. \quad (3)$$

The functional integral has been used and the quark fields are regulated by proper time method[16]. A review of this method has been given by Ball[15]. This integral can be done in Euclid space(forget photon and W boson first)

$$\mathcal{L}_E^M = \log \det \mathcal{D}, \quad (4)$$

where

$$\mathcal{D} = \not{D} - i\not{p} - i\not{p}\gamma_5 + mu. \quad (5)$$

The eq.(4) can be written in two parts

$$\begin{aligned} \mathcal{L}_E^M &= \mathcal{L}_{RE} + \mathcal{L}_{IM}, \\ \mathcal{L}_{RE} &= \frac{1}{2} \log \det (\mathcal{D}^\dagger \mathcal{D}), \quad \mathcal{L}_{IM} = \frac{1}{2} \log \det (\mathcal{D} / \mathcal{D}^\dagger) \end{aligned} \quad (6)$$

where

$$\mathcal{D}^\dagger = -\not{\partial} + i\not{p} - i\not{p}\gamma_5 + m\hat{u}, \quad \hat{u} = \exp(-i)\gamma_5(\tau_i\pi_i + \eta). \quad (7)$$

From this effective lagrangian it can be seen that the physical processes with normal party are described by \mathcal{L}_{RE} and the ones with abnormal party are described by \mathcal{L}_{IM} . In terms of Schwenger's proper time method[16] we have

$$\mathcal{L}_{RE} = \frac{1}{2} \int d^4x Tr \int_0^\infty \frac{d\tau}{\tau} e^{-\mathcal{D}^\dagger \mathcal{D}}, \quad (8)$$

where the trace is taken in color, flavor, and Lorentz space. Inserting a complete set of plane wave and subtracting the divergence at $\tau = 0$, we obtain

$$\mathcal{L}_{RE} = \frac{1}{2} \frac{1}{\delta^D(0)} \int d^Dx \frac{d^Dp}{(2\pi)^D} Tr \int_0^\infty \frac{d\tau}{\tau} (e^{-\tau \mathcal{D}'^\dagger \mathcal{D}'} - e^{-\Delta_0}) \delta^D(x - y)|_{y \rightarrow x} \quad (9)$$

where

$$\begin{aligned} \mathcal{D}' &= \not{\partial} + i\not{p} - i\not{p} - i\not{p}\gamma_5 + mu \\ \mathcal{D}'^\dagger &= -\not{\partial} - i\not{p} + i\not{p} - i\not{p}\gamma_5 + m\hat{u}, \\ \Delta_0 &= p^2 + m^2. \end{aligned} \quad (10)$$

In ref.[15], the Seeley-DeWitt coefficients have been used to evaluate the expansion series of eq.(9). In this paper we use dimensional regularization. After completing the integration over τ the lagrangian \mathcal{L}_{RE} reads

$$\mathcal{L}_{RE} = \frac{1}{2} \int d^Dx \frac{d^Dp}{(2\pi)^D} \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{(p^2 + m^2)^n}$$

$$Tr\{(\not{\partial} - i\not{p} + i\not{p}\gamma_5)(\not{\partial} - i\not{p} - i\not{p}\gamma_5) + 2ip \cdot (\partial - iv - ia\gamma_5) + m\not{D}u\}, \quad (11)$$

where $\not{D}u = \gamma^\mu D_\mu u$ and

$$D_\mu u = \partial_\mu u - i[v_\mu, u] + i\{a_\mu, u\}. \quad (12)$$

To the fourth order in covariant derivatives in Minkofsky space the lagrangian takes following form

$$\begin{aligned} \mathcal{L}_{RE} = & \frac{N_c}{(4\pi)^2} m^2 \frac{D}{4} \Gamma(2 - \frac{D}{2}) Tr D_\mu U D^\mu U^\dagger \\ & - \frac{1}{3} \frac{N_c}{(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{2}) \{2\omega_{\mu\nu}\omega^{\mu\nu} + Tr \rho_{\mu\nu}\rho^{\mu\nu} + 2f_{\mu\nu}f^{\mu\nu} + Tra_{\mu\nu}a^{\mu\nu}\} \\ & + \frac{i}{2} \frac{N_c}{(4\pi)^2} Tr \{D_\mu U D_\nu U^\dagger + D_\mu U^\dagger D_\nu U\} \rho^{\nu\mu} \\ & + \frac{i}{2} \frac{N_c}{(4\pi)^2} Tr \{D_\mu U^\dagger D_\nu U - D_\mu U D_\nu U^\dagger\} a^{\nu\mu} \\ & + \frac{N_c}{6(4\pi)^2} Tr D_\mu D_\nu U D^\mu D^\nu U^\dagger \\ & - \frac{N_c}{12(4\pi)^2} Tr \{D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger + D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U - D_\mu U D_\nu U^\dagger D^\mu U D^\nu U^\dagger\} \\ & + \frac{1}{2} m_0^2 (\omega_\mu \omega_\mu + \rho_\mu^i \rho^{i\mu} + a_\mu^i a^{i\mu} + f_\mu f^\mu), \end{aligned} \quad (13)$$

where

$$D_\mu U = \partial_\mu U - i[\rho_\mu, U] + i\{a_\mu, U\},$$

$$D_\mu U^\dagger = \partial_\mu U^\dagger - i[\rho_\mu, U^\dagger] - i\{a_\mu, U^\dagger\},$$

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu,$$

$$f_{\mu\nu} = \partial_\mu f_\nu - \partial_\nu f_\mu,$$

$$\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i[\rho_\mu, \rho_\nu] - i[a_\mu, a_\nu],$$

$$a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, \rho_\nu] - i[\rho_\mu, a_\nu],$$

$$D_\nu D_\mu U = \partial_\nu(D_\mu U) - i[\rho_\nu, D_\mu U] + i\{a_\nu, D_\mu U\},$$

$$D_\nu D_\mu U^\dagger = \partial_\nu(D_\mu U^\dagger) - i[\rho_\nu, D_\mu U^\dagger] - i\{a_\nu, D_\mu U^\dagger\}.$$

There are correspondence between the schemes of regularization used in this paper and in ref.[15]. Upon this correspondence and transforming the formalism of ref.[15] to Minkofsky space, it can be found that this formalism(13) is the same with the one presented in ref.[15].

The imaginary lagrangian(6) describes the physical processes with abnormal party. It should be the generalized Wess-Zumino lagrangian. We do not use the method of path integral to find \mathcal{L}_{IM} and we evaluate the terms of \mathcal{L}_{IM} , which can be used to study physical processes, in Minkofsky space directly in this paper.

Defining physical meson fields

In eq.(13) there are divergences. The theory studied in this paper is an effective theory and it is not renormalizable. In order to build a physical effective meson theory, the introduction of a cut-off to the theory is necessary and the cut-off will be determined in this theory. We

define

$$\frac{F^2}{16} = \frac{N_c}{(4\pi)^2} m^2 \frac{D}{4} \Gamma(2 - \frac{D}{2}), \quad (14)$$

$$g^2 = \frac{8}{3} \frac{N_c}{(4\pi)^2} \frac{D}{4} \Gamma(2 - \frac{D}{2}) = \frac{1}{6} \frac{F^2}{m^2}. \quad (15)$$

The relationship between the cut-off and F^2 , g will be explored. From the kinetic terms of meson fields in eq.(13) we can see that the meson fields in eq.(13) are not physical. The physical meson fields can be defined in following ways that make the corresponding kinetic terms in the standard form.

$$\begin{aligned} \pi &\rightarrow \frac{2}{f_\pi} \pi, & \eta &\rightarrow \frac{2}{f_\eta} \eta, \\ \rho &\rightarrow \frac{1}{g} \rho, & \omega &\rightarrow \frac{1}{g} \omega, \end{aligned} \quad (16)$$

where f_π and f_η are pion and η decay constants, in chiral limit we take $f_\pi = f_\eta$. Use these substitutions the physical masses of ρ and ω mesons are defined as

$$m_\rho^2 = m_\omega^2 = \frac{1}{g^2} m_0^2. \quad (17)$$

We can also make the same transformation to a_1 and f_1 fields

$$a_\mu^i \rightarrow \frac{1}{g} a_\mu^i, \quad f_\mu \rightarrow \frac{1}{g} f_\mu. \quad (18)$$

However, there are other factors for the normalizations of axial-vector fields. In eq.(13) there are mixing between a_μ^i and $\partial_\mu \pi_i$, f_μ and $\partial_\mu \eta$. In chiral limit the mixing

$$\frac{F^2}{2g} \partial_\mu \pi_i a_i^\mu.$$

comes from the first term of eq.(13). The transformation

$$a_\mu^i \rightarrow a_\mu^i - c \partial_\mu \pi^i \quad (19)$$

has been used to erase the mixing. In chiral limit, c has been determined by cancelling the mixing term

$$c = \frac{\frac{F^2}{2g}}{m_\rho^2 + \frac{F^2}{g^2}}. \quad (20)$$

There is similar mixing term between f_μ and $\partial_\mu \eta$ and the transformation

$$f_\mu \rightarrow f_\mu - c \partial_\mu \eta$$

is used to cancel the mixing term. In chiral limit, c of this formula is the same with eq.(20).

From the term

$$\frac{N_c}{6(4\pi)^2} \text{Tr} D_\nu D_\mu U D^\nu D^\mu U^\dagger$$

of the lagrangian(13), in chiral limit another term related to the normalization of a_μ^i field has been found, which can be written as

$$\frac{1}{8\pi^2 g^2} (\partial_\mu a_\nu^i - \partial_\nu a_\mu^i) (\partial^\mu a^{i\nu} - \partial^\nu a^{i\mu}). \quad (21)$$

Combining this term with the kinetic term of a_μ^i in eq.(13), the physical a_μ^i field has been defined as

$$a_\mu^i \rightarrow \frac{1}{g} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} a_\mu^i. \quad (22)$$

In the same way, we obtain the physical f_μ field

$$f_\mu \rightarrow \frac{1}{g} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} f_\mu. \quad (23)$$

After the transformations(19), in order to make the kinetic term of pion in the standard form, in the chiral limit following equation must be satisfied

$$\frac{F^2}{8} \left(1 - \frac{2c}{g}\right)^2 + \frac{1}{2} m_\rho^2 c^2 = \frac{f_\pi^2}{8}. \quad (24)$$

The eq.(24) makes the kinetic term of η meson field in standard form. Eqs.(20,24) can be simplified as

$$c = \frac{f_\pi^2}{2g m_\rho^2}, \quad (25)$$

$$\frac{F^2}{f_\pi^2} \left(1 - \frac{2c}{g}\right) = 1. \quad (26)$$

New mass formulas of vector mesons and its chiral partners

In lagrangian(1) that vector mesons and axial-vector mesons are chiral partners. However, it can be seen from eqs.(19,22) that vector and axial-vector meson fields behave differently. Due to eqs.(22,23), in the couplings of axial-vector fields to others there is an additional factor $(1 - 1/2\pi^2 g^2)^{-1/2}$.

The physical masses of vector mesons are defined by eq.(17). For the masses of axial-vector mesons there are three contributors: the mass term in the lagrangian(1); the contribution of the first term of the lagrangian(13), which is $\frac{F^2}{g^2}$; the normalization factor $(1 - \frac{1}{2\pi^2 g^2})^{-1}$.

Put all these three factors together the a_1 mass has been found to be

$$(1 - \frac{1}{2\pi^2 g^2})m_a^2 = \frac{F^2}{g^2} + m_\rho^2. \quad (27)$$

In the same way, we obtain the mass formula of f_1 meson

$$(1 - \frac{1}{2\pi^2 g^2})m_f^2 = \frac{F^2}{g^2} + m_\omega^2. \quad (28)$$

If we ignore the mass difference of ρ and ω mesons from these two mass formulas we obtain

$$m_f = m_a. \quad (29)$$

The deviation of this relation from physical values($m_a = 1.26GeV$ and $m_f = 1.285GeV$) is about 2%.

In chiral limit there are three parameters in this theory, which can be chosen as g , f_π , and m_ρ . Take f_π and m_ρ as inputs and choose

$$g = 0.35 \quad (30)$$

to get a better fits. The couplings in all the physical processes described by \mathcal{L}_{RE} and \mathcal{L}_{IM} are fixed by g and c . This is the universality of coupling in this theory.

Vector meson dominance(VMD)

Vector meson dominance(VMD) has been revealed from this theory. From eq.(1) it can be

seen that except the kinetic term of photon, photon and vector mesons always appear in the combinations

$$\frac{1}{g}\rho_\mu^0 + \frac{1}{2}eA_\mu, \quad \frac{1}{g}\omega_\mu + \frac{1}{6}eA_\mu. \quad (31)$$

Therefore, the interaction of photon with other fields can be found from the interactions of ρ^0 or ω with other fields by using the substitutions

$$\begin{aligned} \rho_\mu^0 &\rightarrow \frac{1}{2}egA_\mu, \\ \omega_\mu &\rightarrow \frac{1}{6}egA_\mu. \end{aligned} \quad (32)$$

Incorporating photon field into lagrangian(13), from the kinetic terms of ρ^0 and ω mesons in lagrangian(13) we obtain

$$\begin{aligned} &-\frac{1}{4}\{\partial_\mu(\rho_\nu^0 + \frac{1}{2}e_0gA_\nu) - \partial_\nu(\rho_\mu^0 + \frac{1}{2}e_0gA_\mu)\}^2, \\ &-\frac{1}{4}\{\partial_\mu(\omega_\nu + \frac{1}{6}e_0gA_\nu) - \partial_\nu(\omega_\mu + \frac{1}{6}e_0gA_\mu)\}^2. \end{aligned} \quad (33)$$

In order to make the kinetic term of photon field in standard form, it is needed to redefine the photon field and the charge to be

$$A_\mu \rightarrow (1 + \frac{5e_0^2g^2}{18})^{-\frac{1}{2}}A_\mu, \quad e_0 \rightarrow e(1 + \frac{5e_0^2g^2}{18})^{\frac{1}{2}}, \quad e_0A_\mu \rightarrow eA_\mu. \quad (34)$$

From eq.(33) the couplings between photon and vector mesons have been obtained

$$\begin{aligned} &-\frac{1}{2}\frac{e}{f_\rho}F_{\mu\nu}(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu), \\ &-\frac{1}{2}\frac{e}{f_\omega}F_{\mu\nu}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu), \end{aligned} \quad (35)$$

where

$$\frac{1}{f_\rho} = \frac{1}{2}g, \quad \frac{1}{f_\omega} = \frac{1}{6}g. \quad (36)$$

The ratio of $\frac{1}{f_\rho}$ to $\frac{1}{f_\omega}$ is $1 : \frac{1}{3}$, that is the same with quark model. The comparison between theoretical and experimental values of f_ρ and f_ω can be found in Table II. The photon-vector meson couplings shown by eqs.(35) are just the ones proposed in ref.[18]. On the other hand, there are interactions between ρ and ω mesons with other mesons, in general, these interactions can be written as

$$\rho^{i\mu} j_\mu^i + \omega^\mu j_\mu^\omega.$$

Therefore, besides the direct coupling of photon and ρ meson(35) another type of interaction between photon and other mesons can be found by the substitution(32)

$$\frac{e}{f_\rho} A^\mu j_\mu^0 + \frac{e}{f_\omega} A_\mu j_\mu^\omega.$$

The complete expression of the interaction between isovector photon and mesons is

$$\frac{e}{f_\rho} \left\{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu \rho_\nu^0 - \partial_\nu \rho_\mu^0) + A^\mu j_\mu^0 \right\}. \quad (37)$$

This is the exact expression of VMD proposed by Sakurai[19]. In the same way the isoscalar

VMD has been found

$$\frac{e}{f_\omega} \left\{ -\frac{1}{2} F^{\mu\nu} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) + A^\mu j_\mu^\omega \right\}. \quad (38)$$

The cause obtaining the explicit expression of VMD in this theory can be manifested in another way. It is well known that in *QCD* the electric current takes the form of

$$\bar{\psi}Q\gamma_\mu\psi = \frac{1}{2}\bar{\psi}\tau_i\gamma_\mu\psi + \frac{1}{6}\bar{\psi}\gamma_\mu\psi$$

which is in the lagrangian(1). The electric current $\bar{\psi}Q\gamma_\mu\psi$ can be bosonized in this theory. In ref.[20] we have developed a method to find the effective currents in the case that only pseudoscalar fields are taken as background fields. In this paper this method has been generalized to include vector and axial-vector mesons. From the lagrangian(1) the equation satisfied by the quark propagator has been obtained

$$\{i\cancel{\partial} + \frac{1}{g}\cancel{p}(x) + \frac{1}{g'_a}\cancel{p}(x)\gamma_5 - mu(x)\}s_F(x, y) = \delta^4(x - y), \quad (39)$$

where $g'_a = g(1 - \frac{1}{2\pi^2g^2})^{\frac{1}{2}}$. In momentum picture there is

$$s_F(x, y) = \frac{1}{(2\pi)^4} \int d^4y e^{-ip(x-y)} s_F(x, p). \quad (40)$$

The eq.(39) becomes

$$\{i\cancel{\partial} + \cancel{p} + \frac{1}{g}\cancel{p}(x) + \frac{1}{g'_a}\cancel{p}(x)\gamma_5 - mu(x)\}s_F(x, p) = 1. \quad (41)$$

Eq.(41) has been solved

$$s_F(x, p) = s_F^0 \sum_{n=0}^{\infty} (-)^n (\{i\cancel{\partial} + \frac{1}{g}\cancel{p}(x) + \frac{1}{g'_a}\cancel{p}(x)\gamma_5\}s_F^0)^n \quad (42)$$

where

$$s_F^0 = -\frac{\not{p} - m\hat{u}}{p^2 - m^2}. \quad (43)$$

In terms of eqs.(40,42) the bosonization of quark electric current has been done in following way

$$\begin{aligned} \langle \bar{\psi}(x)Q\gamma_\mu\psi(x) \rangle &= \frac{1}{2} \langle \bar{\psi}(x)\tau_3\gamma_\mu\psi(x) \rangle + \frac{1}{6} \langle \bar{\psi}(x)\gamma_\mu\psi(x) \rangle, \\ \langle \bar{\psi}(x)\tau_3\gamma_\mu\psi(x) \rangle &= -iTr\tau_3\gamma_\mu s_F(x, x), \quad \langle \bar{\psi}(x)\gamma_\mu\psi(x) \rangle = -iTr\gamma_\mu s_F(x, x). \end{aligned} \quad (44)$$

The leading terms of eq.(44) have been found at $n = 3$ and the effective currents take following forms

$$\langle \bar{\psi}\tau_3\gamma_\mu\psi \rangle = g\partial^2\rho_\mu^0 + gj_\mu^0, \quad \langle \bar{\psi}\gamma_\mu\psi \rangle = g\partial^2\omega_\mu + gj_\mu^\omega. \quad (45)$$

In obtaining the first term of eqs.(45), eq.(15) has been used. In eq.(45) j_μ^0 and j_μ^ω have been defined as the rest parts of the currents and j_μ^ω will be evaluated explicitly below. From eq.(1) it can be seen that j_μ^0 couples to ρ_μ^0 and j_μ^ω couples to ω . Therefore, these two currents are the currents mentioned in eqs.(37,38). After getting rid of total derivative terms we have

$$\begin{aligned} eA^\mu \langle \bar{\psi}Q\gamma_\mu\psi \rangle &= \frac{1}{f_\rho} \left\{ -\frac{1}{2}F^{\mu\nu}(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu) + A^\mu j_\mu^0 \right\} \\ &+ \frac{1}{f_\omega} \left\{ -\frac{1}{2}F^{\mu\nu}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu) + A^\mu j_\mu^\omega \right\}. \end{aligned} \quad (46)$$

This the same with eqs.(37,38).

The decays of $\rho \rightarrow \pi\pi$, $\omega \rightarrow \pi\pi$, and KSFR sum rule

In this theory pion is associated with γ_5 (see eq.(1)) and only even number of γ_5 involved in $\rho\pi\pi$ vertex. Therefore, the vertex of $\rho\pi\pi$ can be found from eq.(13), which is

$$\mathcal{L}_{\rho\pi\pi} = \frac{2}{g}\epsilon_{ijk}\rho_i^\mu\pi_j\partial_\mu\pi_k + \frac{2}{\pi^2gf_\pi^2}[4\pi^2c^2 - (1 - \frac{2c}{g})^2]\epsilon_{ijk}\rho_i^\mu\partial_\nu\pi_j\partial_{\mu\nu}\pi_k. \quad (47)$$

In deriving eq.(47), eq.(26) has been used. For the decay of $\rho \rightarrow \pi\pi$ eq.(47) becomes the

$$\begin{aligned} \mathcal{L}_{\rho\pi\pi} &= f_{\rho\pi\pi}\epsilon_{ijk}\rho_i^\mu\pi_j\partial_\mu\pi_k, \\ f_{\rho\pi\pi} &= \frac{2}{g}\{1 + \frac{m_\rho^2}{2\pi^2f_\pi^2}[(1 - \frac{2c}{g})^2 - 4\pi^2c^2]\}. \end{aligned} \quad (48)$$

The choice of $g = 0.35$ makes

$$f_{\rho\pi\pi} = \frac{2}{g}. \quad (49)$$

Using eq.(48) we obtain

$$\Gamma(\rho \rightarrow \pi\pi) = \frac{f_{\rho\pi\pi}^2}{48\pi}m_\rho(1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}} = 135MeV. \quad (50)$$

The experimental value is 151MeV. The deviation is about 10%. The KSFR sum rule[5]

$$g_{\rho\gamma} = \frac{1}{2}f_{\rho\pi\pi}f_\pi \quad (51)$$

is the result of current algebra and PCAC. From eq.(35) we have

$$g_{\rho\gamma} = \frac{1}{2}gm_\rho^2. \quad (52)$$

Substituting eqs.(49,52) into the KSFR sum rule we obtain

$$g^2 = 2 \frac{f_\pi^2}{m_\rho^2}, \quad g = 0.342. \quad (53)$$

Comparing the value of g with (30) it can be seen that KSFR sum rule is satisfied pretty well.

In chiral limit the mixing between ω and ρ is caused by electromagnetic interaction. The lagrangian of this mixing is

$$\mathcal{L}_i = -\frac{1}{4}egF^{\mu\nu}(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu) - \frac{1}{12}egF^{\mu\nu}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu). \quad (54)$$

The mixing angle has been found from eq.(54)

$$\begin{aligned} \sin 2\theta &= \frac{\frac{2\pi}{3}\alpha g^2 \bar{m}_\rho^2}{m_\omega^2 - m_\rho^2}, \\ \theta &= 1.74^0, \end{aligned} \quad (55)$$

where $\bar{m}_\rho^2 = \frac{1}{2}(m_\rho^2 + m_\omega^2)$. The decay width is

$$\Gamma(\omega \rightarrow \pi\pi) = \sin^2\theta \Gamma(\rho \rightarrow \pi\pi) \frac{p^{*3}}{p^3} \frac{m_\rho^2}{m_\omega^2} = 0.136 MeV, \quad (56)$$

where p^* is the momentum of pion when the mass of ρ is m_ω and p is the momentum of pion when the mass of ρ is really m_ρ . The experimental value is $0.186(1 \pm 0.15)MeV$.

Pion form factor

According to VMD(eq.(37)), the vertex of $\pi\pi\gamma$ consists of two parts: direct coupling and the

indirect coupling through a ρ meson. The lagrangian of direct coupling can be found either from the eq.(13) or by substituting $\rho^0 \rightarrow \frac{e}{f_\rho} A$ in eq.(48)

$$\mathcal{L}_{\pi\pi\gamma} = e \frac{f_{\rho\pi\pi}}{f_\rho} \epsilon_{3jk} A^\mu \pi_j \partial_\mu \pi_k. \quad (57)$$

Due to the coupling $-\frac{1}{2} \frac{e}{f_\rho} F_{\mu\nu} (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu)$ the indirect coupling of $\pi\pi\gamma$ is proportional to q^2 (q is the momentum of photon), therefore the charge normalization of π^+ is satisfied by $\frac{f_{\rho\pi\pi}}{f_\rho} = 1$. From the lagrangian

$$\mathcal{L} = -\frac{1}{2} \frac{e}{f_\rho} \{F^{\mu\nu} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\} + \mathcal{L}_{\pi\pi\gamma} + \mathcal{L}_{\rho\pi\pi}. \quad (58)$$

The pion form factor has been obtained

$$F_\pi(q^2) = \frac{1}{1 - \frac{q^2}{m_\rho^2}}. \quad (59)$$

VMD results ρ pole in pion form factor[21]. The radius of pion is

$$\sqrt{\langle r^2 \rangle_\pi} = 0.63 fm. \quad (60)$$

The experimental value is $0.663 \pm 0.023 fm$ [22].

Decays of $a_1 \rightarrow \rho\pi$ and $\pi\gamma$

The decay $a_1 \rightarrow \rho\pi$ is a process with normal parity. In chiral limit this vertex has been found from eq.(13)

$$\mathcal{L}_{a_1 \rightarrow \rho\pi} = \epsilon_{ijk} \{A a_i^\mu \rho_{j\mu} \pi_k + B a_i^\mu \rho_j^\nu \partial_{\mu\nu} \pi_k\},$$

$$\begin{aligned}
A &= \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left\{ \frac{F^2}{g^2} + \frac{m_a^2}{2\pi^2 g^2} - \frac{2c}{g} (p_\pi \cdot p_\rho + p_\pi \cdot p_a) - \frac{3}{2\pi^2 g^2} \left(1 - \frac{2c}{g}\right) p_\pi \cdot p_\rho \right\} \\
&= \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left\{ \frac{F^2}{g^2} + \frac{m_a^2}{2\pi^2 g^2} - \left[\frac{2c}{g} + \frac{3}{4\pi^2 g^2} \left(1 - \frac{2c}{g}\right) \right] (m_a^2 - m_\rho^2) \right\} \\
&= \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} (m_a^2 - m_\rho^2) \left(1 - \frac{2c}{g}\right) \left(1 - \frac{3}{4\pi^2 g^2}\right), \\
B &= -\frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \frac{1}{2\pi^2 g^2} \left(1 - \frac{2c}{g}\right). \tag{61}
\end{aligned}$$

The three expressions of A in eq.(61) have different uses in this paper and in obtaining the last two expressions of A eq.(27) has been used. The width of the decay has been calculated and is 326MeV which is comparable with data[17]. From eq.(61) it can be seen that there are s-wave and d-wave in this decay. The ratio[23] of these two waves obtained in this theory is

$$\frac{s}{d} = -\frac{1}{3} p_\pi^2 \frac{\frac{A}{m_\rho(m_\rho + E_\rho)} - B \frac{m_a}{m_\rho}}{A \left(1 + \frac{1}{3} \frac{p_\pi^2}{m_\rho(m_\rho + E_\rho)}\right) - \frac{B}{3} \frac{m_a}{m_\rho} p_\pi^2} = -0.097. \tag{62}$$

The quark model[24] predicts that $\frac{d}{s} = -0.15$. Experimental value is -0.11 ± 0.02 [25].

The vertex of $a_1 \rightarrow \pi\gamma$ has been obtained by using substitution(32) in eq.(61)

$$\begin{aligned}
\mathcal{L}_{a_1 \rightarrow \pi\gamma} &= -\epsilon_{3jk} \{ A a_j^\mu A_\mu \pi_k + B a_j^\mu A^\nu \partial_{\mu\nu} \pi_k \}, \\
A &= \frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left\{ \frac{F^2}{g^2} + \frac{m_a^2}{2\pi^2 g^2} - \left[\frac{2c}{g} + \frac{3}{4\pi^2 g^2} \left(1 - \frac{2c}{g}\right) \right] (m_a^2 - q^2) \right\}, \tag{63}
\end{aligned}$$

where q is photon momentum and A is obtained from eq.(61). Before presenting the numerical result, let's prove the current conservation in the case of real photon. In order to have

current conservation following equation should be satisfied

$$A(q^2 = 0) = \frac{1}{2}m_a^2 B. \quad (64)$$

The left hand of this equation can be written as

$$\frac{2}{f_\pi} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left\{ \left(1 - \frac{2c}{g}\right) \left(1 - \frac{1}{2\pi^2 g^2}\right) m_a^2 - m_\rho^2 - \frac{1}{4\pi^2 g^2} \left(1 - \frac{2c}{g}\right) m_a^2 \right\}. \quad (65)$$

Using the mass formula(27), and the expression of c(25) it can be found that the left hand of the equation(65) is just $\frac{1}{2}m_a^2 B$. Therefore, the current conservation in the process of $a_1 \rightarrow \pi\gamma$ is satisfied. The decay width of $a_1 \rightarrow \pi\gamma$ has been computed and is 252keV. The experimental value is 640 ± 246 keV[26].

Use \mathcal{L}_{RE} the decay width of $a_1 \rightarrow 3\pi$ can be calculated. It is found that the branch ratio is 5×10^{-4} which is consistent with data 0.003 ± 0.003 [27].

Revisit of Weinberg's sum rules

From chiral symmetry, current algebra, and VMD Weinberg has found the first sum rule[4]

$$\frac{g_\rho^2}{m_\rho^2} - \frac{g_a^2}{m_a^2} = \frac{1}{4}f_\pi^2, \quad (66)$$

where

$$\begin{aligned} < 0 | \bar{\psi} \frac{\tau_i}{2} \gamma_\mu \psi | \rho_\lambda^j > = g_\rho \epsilon_\mu^\lambda, \\ < 0 | \bar{\psi} \frac{\tau_i}{2} \gamma_\mu \gamma_5 \psi | \rho_\lambda^j > = g_a \epsilon_\mu^\lambda. \end{aligned} \quad (67)$$

Assuming an additional condition[4], Weinberg's second sum rule is obtained

$$g_a = g_\rho. \quad (68)$$

Eqs.(66,68) and KSFR sum rule together lead to $m_a^2 = 2m_\rho^2$ which is not in good agreement with present value of m_a . The theory presented in this paper can be considered as a realization of chiral symmetry, current algebra, and VMD. In this theory, the isovector vector and axial-vector currents in eq.(1) are the same with the ones in eqs.(67). Therefore, g_ρ and g_a can be evaluated explicitly. Using eqs.(40,42) following expressions have been found

$$\begin{aligned} \langle \bar{\psi} \frac{\tau_i}{2} \gamma_\mu \gamma_5 \psi \rangle &= -\frac{1}{2} g m_\rho^2 \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} a_\mu^i + \dots, \\ \langle \bar{\psi} \frac{\tau_i}{2} \gamma_\mu \psi \rangle &= -\frac{1}{2} g m_\rho^2 \rho_\mu^i + \dots \end{aligned} \quad (69)$$

We obtain

$$\begin{aligned} g_\rho &= -\frac{1}{2} g m_\rho^2, \\ g_a &= -\frac{1}{2} g m_\rho^2 \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}}. \end{aligned} \quad (70)$$

The relation(68) is not satisfied and $m_a^2 = 2m_\rho^2$ is not confirmed by this theory. This is the reason why the mass relation(27) obtained in this paper is not the same with the one of ref.[4]. However, Weinberg's first sum rule(66) only depends on chiral symmetry, VMD, and current algebra. Therefore, it should be achieved in this theory. Substituting g_ρ and g_a (70)

into the left hand side of eq.(66), we obtain

$$\frac{g^2}{4}m_\rho^2\{1 - \frac{m_\rho^2}{m_a^2}(1 - \frac{1}{2\pi^2g^2})^{-1}\}. \quad (71)$$

Substituting the mass formula(27) and eq.(26) into eq.(71), indeed Weinberg's first sum rule is satisfied. The factor $(1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}}$ plays important role in this theory.

In ref.[28] two new formulas of the amplitude of $a_1 \rightarrow \rho\pi$ in the limit of $p_\pi \rightarrow 0$ have been found from the Ward identity found by Weinberg[4] and VMD

$$g_a f_\pi A(m_\rho^2) = 2g_\rho(m_a^2 - m_\rho^2), \quad g_\rho f_\pi A(m_a^2) = 2g_a(m_a^2 - m_\rho^2). \quad (72)$$

It needs to check if these two relations are satisfied in this theory. According to ref.[28], in the limit of $p_\pi = 0$ the amplitude A(61) of $a_1 \rightarrow \rho\pi$ can be written as

$$A(k^2) = \frac{2}{f_\pi} \left\{ \frac{F^2}{g^2} + \frac{k^2}{2\pi^2g^2} \right\} (1 - \frac{1}{2\pi^2g^2})^{-\frac{1}{2}}. \quad (73)$$

Using eq.(73) and the mass formula(27), the two relations(72) are indeed satisfied.

The decays of $\tau \rightarrow \rho\nu$ and $a_1\nu$

The decay rates of $\tau \rightarrow \rho\nu$ and $a_1\nu$ can be calculated in terms of the two matrix elements(67,70)

$$\begin{aligned} \Gamma(\tau \rightarrow \rho\nu) &= \frac{G^2}{8\pi} \cos^2\theta g_\rho^2 \frac{m_\tau^3}{m_\rho^2} (1 - \frac{m_\rho^2}{m_\tau^2})^2 (1 + 2\frac{m_\rho^2}{m_\tau^2}) = 4.84 \times 10^{-13} GeV, \\ \Gamma(\tau \rightarrow a_1\nu) &= \frac{G^2}{8\pi} \cos^2\theta g_a^2 \frac{m_\tau^3}{m_a^2} (1 - \frac{m_a^2}{m_\tau^2})^2 (1 + 2\frac{m_a^2}{m_\tau^2}) = 1.56 \times 10^{-13} GeV. \end{aligned} \quad (74)$$

The experimental values are $(0.495 \pm 0.023)10^{-12} GeV$ and $(2.42 \pm 0.76)10^{-13} GeV$ for $\tau \rightarrow \rho\nu$ and $\tau \rightarrow a_1\nu$ respectively.

These calculations can also be done in terms of the effective lagrangian(13) of mesons, in which the couplings between the mesons and W bosons are determined. In chiral limit, the $\pi - W$ coupling has been found from the first term of the lagrangian(13)

$$-\frac{F^2}{4f_\pi}(1 - \frac{2c}{g})g_w\partial_\mu\pi_iW_i^\mu = -\frac{f_\pi}{4}g_w\partial_\mu\pi_iW_i^\mu. \quad (75)$$

From π_{l2} decay, f_π has been determined to be 186MeV.

Like photon, from the lagrangian(1) it can be seen that W bosons always appear in the combinations either $\rho_\mu + \frac{g_w}{4}g(\tau_1W_\mu^1 + \tau_2W_\mu^2)$ or $a_\mu - \frac{g_w}{4}g'_a(\tau_1W_\mu^1 + \tau_2W_\mu^2)$. The W boson fields of eq.(1) are needed to be normalized.

$$W \rightarrow (1 + 2g^2\frac{g_w^2}{4})^{-\frac{1}{2}}W,$$

$$g_w \rightarrow (1 + 2g^2\frac{g_w^2}{4})^{\frac{1}{2}}g_w, \quad g_wW_\mu \rightarrow g_wW_\mu.$$

Like VMD, the coupling of $\rho - W$ has been found

$$-\frac{g_w}{4}\frac{1}{2}g(\partial_\mu\rho_\nu^i - \partial_\nu\rho_\mu^i)(\partial^\mu W^{i\nu} - \partial^\nu W^{i\mu}). \quad (76)$$

When ρ is on mass shell the coupling becomes g_ρ . Of course, like VDM, there is direct coupling between W boson and other mesons

$$\frac{g_w}{4}\frac{1}{2}gW_\mu^iJ^{i\mu}. \quad (77)$$

The axial-vector part of the interactions of W boson with mesons has been found

$$\begin{aligned}
& -\frac{g_w}{4}g\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}\left(\frac{F^2}{g^2}+\frac{1}{2\pi^2g^2}m_a^2\right)a_\mu^iW^{i\mu}-\frac{g_w}{4}f_\pi W_{i\mu}\partial_\mu\pi_i \\
& +\frac{g_w}{4}\frac{1}{2}g\left(1-\frac{1}{2\pi^2g^2}\right)^{-\frac{1}{2}}(\partial_\mu a_\nu^i-\partial_\nu a_\mu^i)(\partial^\mu W^{i\nu}-\partial^\nu W^{i\mu})-\frac{g_w}{4}\frac{1}{2}gW_\mu^i j_a^{i\mu},
\end{aligned} \tag{78}$$

where $j_a^{i\mu}$ is defined as isovector axial-vector current and a_1 fields couple to this current.

Using the mass formula(27), it can be seen from eq.(78) that the coupling $a_1 - W$ is just $g_a(70)$.

$\pi\pi$ scattering and determination of parameters of CPT

The pion is nearly Goldstone boson associated with the dynamically broken $SU(2)_L \times SU(2)_R$ chiral symmetry which is a symmetry of QCD in the limit $m_{u,d} \rightarrow 0$. $\pi\pi$ scattering has been studied by Weinberg, using nonlinear chiral lagrangian[29]. The modern study of $\pi\pi$ scattering utilizes chiral perturbation theory[2,30]. In present theory the $\pi\pi$ scattering is related to even number of γ_5 , hence the lagrangian of this process can be found from eq.(13). As discussed in ref.[31] the pion mass term can be introduced to the theory by adding the quark mass term $\bar{\psi}M\psi$ (M is the quark mass matrix) to the lagrangian(1). According to ref.[31] the pion mass term obtained from the quark mass term is

$$\frac{1}{8}f_\pi^2 m_\pi^2 Tr(U - 1). \tag{79}$$

The $\pi_{j_1}(k_1) + \pi_{j_2}(k_2) \rightarrow \pi_{i_1}(p_1)\pi_{i_2}(p_2)$ scattering amplitudes are written as

$$T_{j_1 j_2, i_1 i_2} = A(s, t, u)\delta_{i_1 i_2}\delta_{j_1 j_2} + A(t, s, u)\delta_{i_1 j_1}\delta_{i_2 j_2} + A(u, t, s)\delta_{i_1 j_2}\delta_{i_2 j_1}, \tag{80}$$

where $s = (k_1 + k_2)^2$, $t = (k_1 - p_1)^2$, $u = (k_1 - p_2)^2$. In the frame of center of mass $s = 4m_\pi^2 + 4k^2$, $t = -2k^2(1 - \cos\theta)$, $u = -2k^2(1 + \cos\theta)$, where k is the pion momentum and θ is the scattering angle. The partial wave amplitudes are defined as

$$\begin{aligned} T_l^I(s) &= \frac{1}{64\pi} \int_{-1}^1 d\cos\theta P_l(\cos\theta) T^I(s, t, u), \\ T^0 &= 3A(s, t, u) + A(t, s, u) + A(u, t, s), \\ T^1 &= A(t, s, u) - A(u, t, s), \\ T^2 &= A(t, s, u) + A(u, t, s). \end{aligned} \tag{81}$$

At low energies, the partial wave amplitudes can be expanded in terms of scattering length a_l^I and slope b_l^I

$$ReT_l^I(s) = \left(\frac{k^2}{m_\pi^2}\right)^l (a_l^I + \frac{k^2}{m_\pi^2} b_l^I). \tag{82}$$

The lagrangian of $\pi\pi$ scattering derived from eq.(13) contains two parts: direct coupling and ρ meson exchange. To the leading order of chiral perturbation, the amplitudes obtained from direct coupling(with an index D) are

$$\begin{aligned} A(s, t, u)_D &= \frac{16}{f_\pi^4} \left\{ \frac{1}{3} f_\pi^2 \left(1 - \frac{6c}{g}\right) (2m_\pi^2 + 3k^2) + \frac{c^2}{g^2} \left(\frac{3}{\pi^2} \left(1 - \frac{2c}{g}\right)^2 - 4c^2\right) (6k^4 - 2k^4 \cos^2\theta) \right. \\ &\quad \left. - \frac{4}{(4\pi)^2} \left(1 - \frac{2c}{g}\right)^4 (10k^4 - 2k^4 \cos^2\theta) + \frac{8}{(4\pi)^2} \left(1 - \frac{2c}{g}\right)^2 \left[-\frac{16c}{g} k^4 + 4k^4 + \frac{4c^2}{g^2} k^4 (1 + \cos^2\theta)\right] \right\}, \\ A(t, s, u)_D &= \frac{16}{f_\pi^4} \left\{ \frac{1}{6} f_\pi^2 \left(1 - \frac{6c}{g}\right) (-2m_\pi^2 - 3k^2 + 3k^2 \cos\theta) \right. \\ &\quad \left. + \frac{c^2}{g^2} \left(\frac{3}{\pi^2} \left(1 - \frac{2c}{g}\right)^2 - 4c^2\right) [-3k^4 + k^4 \cos^2\theta - 6k^4 \cos\theta] \right\} \end{aligned}$$

$$\begin{aligned}
& -\frac{4}{(4\pi)^2}\left(1 - \frac{2c}{g}\right)^4[-2k^4 + 2k^4\cos^2\theta - 8k^4\cos\theta] \\
& +\frac{8}{(4\pi)^2}\left(1 - \frac{2c}{g}\right)^2\left[\frac{2c}{g}(-2k^4 - 2k^4\cos^2\theta + 4k^4\cos\theta) + (-k^2 + k^2\cos\theta)^2\right. \\
& \left. +\frac{2c^2}{g^2}(5k^4 + 2k^4\cos\theta + k^4\cos^2\theta)\right]\}.
\end{aligned} \tag{83}$$

The eq.(26) has been used in deriving eqs.(83). The amplitude $A(u,t,s)$ can be obtained by using the substitution of $\cos\theta \rightarrow -\cos\theta$ in $A(t,s,u)$. The amplitudes from ρ exchange have been obtained by using eqs.(48,49)

$$\begin{aligned}
A(s, t, u)_\rho &= \frac{8}{g^2} \frac{2m_\pi^2 + 3k^2 + k^2\cos\theta}{m_\rho^2 + 2k^2 - 2k^2\cos\theta} + \frac{8}{g^2} \frac{2m_\pi^2 + 3k^2 - k^2\cos\theta}{m_\rho^2 + 2k^2 + 2k^2\cos\theta}, \\
A(t, s, u)_\rho &= \frac{16}{g^2} \frac{k^2\cos\theta}{m_\rho^2 - s + im_\rho\Gamma(k)} - \frac{8}{g^2} \frac{2m_\pi^2 + 3k^2 - k^2\cos\theta}{m_\rho^2 + 2k^2 + 2k^2\cos\theta}, \\
A(u, t, s)_\rho &= \frac{-16}{g^2} \frac{k^2\cos\theta}{m_\rho^2 - s + im_\rho\Gamma(k)} - \frac{8}{g^2} \frac{2m_\pi^2 + 3k^2 + k^2\cos\theta}{m_\rho^2 + 2k^2 - 2k^2\cos\theta},
\end{aligned} \tag{84}$$

where $\Gamma(k)$ is the decay width of ρ meson

$$\Gamma(k) = \frac{2}{3\pi g^2} \frac{k^3}{m_\rho^2}. \tag{85}$$

Due to kinematic reason the decay width of ρ meson appears only when the virtual momentum squared of ρ is equal to s .

The scattering lengths and slopes have been found from direct coupling(eqs.(83))

$$\begin{aligned}
a_0^0 &= \frac{5m_\pi^2}{24\pi f_\pi^2} + \frac{2m_\pi^2}{3\pi f_\pi^2} \left(1 - \frac{6c}{g}\right), \\
b_0^0 &= \frac{m_\pi^2}{\pi f_\pi^2} \left(1 - \frac{6c}{g}\right),
\end{aligned}$$

$$\begin{aligned}
a_0^2 &= \frac{m_\pi^2}{12\pi f_\pi^2} - \frac{m_\pi^2}{3\pi f_\pi^2} \left(1 - \frac{6c}{g}\right), \\
b_0^2 &= -\frac{m_\pi^2}{2\pi f_\pi^2} \left(1 - \frac{6c}{g}\right), \\
a_1^1 &= \frac{m_\pi^2}{6\pi f_\pi^2} \left(1 - \frac{6c}{g}\right),
\end{aligned} \tag{86}$$

and from ρ exchange(84) we obtain

$$\begin{aligned}
a_0^0 &= \frac{2m_\pi^2}{\pi g^2 m_\rho^2}, \quad b_0^0 = \frac{3m_\pi^2}{\pi g^2 m_\rho^2}, \quad a_0^2 = -\frac{m_\pi^2}{\pi g^2 m_\rho^2}, \\
b_0^2 &= -\frac{3m_\pi^2}{2\pi g^2 m_\rho^2}, \quad a_1^1 = \frac{m_\pi^2}{2\pi g^2 m_\rho^2}.
\end{aligned} \tag{87}$$

Numerical calculation shows that in these quantities the contribution of ρ exchange is dominant, for instance, the contribution of ρ exchange to a_0^0 is ten more times of the one from direct coupling. Adding eqs.(86,87) together and using eq.(25) we obtain

$$a_0^0 = \frac{7m_\pi^2}{8\pi f_\pi^2}, \quad b_0^0 = \frac{m_\pi^2}{\pi f_\pi^2}, \quad a_0^2 = -\frac{m_\pi^2}{4\pi f_\pi^2}, \quad b_0^2 = -\frac{m_\pi^2}{2\pi f_\pi^2}, \quad a_1^1 = \frac{m_\pi^2}{6\pi f_\pi^2}. \tag{88}$$

These are just the scattering lengths and slopes obtained by Weinberg[29]. In eqs.(86), there are terms with the factor of $\frac{c}{g}$ obtained from the shift $a_\mu \rightarrow a_\mu - c\partial_\mu\pi$. Due to eq.(25) these terms are cancelled by the corresponding terms obtained from ρ exchange. These cancellations result Weinberg's results in this theory. As a matter of fact, the cancellation is the result of chiral symmetry. In the lagrangian(1), there is a term $\frac{1}{2}m_0^2(a_\mu a^\mu + v_\mu v_\mu)$ introduced by chiral symmetry and due to this term, c (see eq.(25)) has m_ρ^2 in the denominator of the expression(25) which leads to the cancellation. All these quantities are only related

to the zeroth and the second orders of derivatives. The scattering lengths and slope a_2^0 , a_0^2 , and b_1^1 have been found from the terms with the derivatives at the fourth order

$$\begin{aligned} b_1^1 &= \frac{m_\pi^4}{6\pi^3 f_\pi^4}(-0.036) + \frac{2m_\pi^4}{\pi g^2 m_\rho^4}, \\ a_2^0 &= \frac{m_\pi^4}{10\pi^3 f_\pi^4}(0.0337) + \frac{4m_\pi^4}{15\pi g^2 m_\rho^4}, \\ a_2^2 &= \frac{m_\pi^4}{10\pi^3 f_\pi^4}(0.0207) - \frac{2m_\pi^4}{15\pi g^2 m_\rho^4}. \end{aligned} \quad (89)$$

In these quantities(89), the contributions of ρ exchange are ten more times of the ones from direct coupling. ρ meson exchange is dominant. The numerical results are listed in Table I.

It is well known that the chiral perturbation theory[2,30] describes $\pi\pi$ scattering pretty well. In principle the parameters of chiral perturbation theory can be calculated by present theory. Besides f_π and m_π there are other two parameters appearing in $\pi\pi$ scattering[30]

$$\mathcal{L}_4 = \frac{\alpha_1}{4} Tr\{\partial_\mu U \partial^\mu U^\dagger\}^2 + \frac{\alpha_2}{4} Tr(\partial_\mu U \partial_\nu U^\dagger) Tr(\partial^\mu U \partial^\nu U^\dagger). \quad (90)$$

The amplitudes T_2^0 and T_2^2 have been determined by \mathcal{L}_4 [30]

$$T_2^0 = \frac{2\alpha_2 + \alpha_1}{15\pi f_\pi^4} (s - 4m_\pi^2)^2, \quad T_2^2 = \frac{2\alpha_1 + \alpha_2}{30\pi f_\pi^4} (s - 4m_\pi^2)^2. \quad (91)$$

From the amplitudes(83,84)(to $O(k^4)$), we obtain

$$T_2^0 = \frac{1}{15\pi f_\pi^4} (s - 4m_\pi^2)^2 (0.00698), \quad T_2^2 = -\frac{1}{30\pi f_\pi^4} (s - 4m_\pi^2)^2 (0.00656). \quad (92)$$

Table 1: Table I The pion scattering lengths and slopes

	Experimental	Theoretical
a_0^0	0.26 ± 0.05	0.16
b_0^0	0.25 ± 0.03	0.18
a_0^2	-0.028 ± 0.012	-0.045
b_0^2	-0.082 ± 0.008	-0.089
a_1^1	0.038 ± 0.002	0.030
b_1^1		5.56×10^{-3}
a_2^0	$(17 \pm 3) \times 10^{-4}$	7.84×10^{-4}
a_2^2	$(1.3 \pm 3) \times 10^{-4}$	-3.53×10^{-4}

The two parameters in eq.(91) have been determined

$$\alpha_1 = -0.0068, \quad \alpha_2 = 0.0070. \quad (93)$$

They are compatible with the values of CPT[30]

$$\alpha_1 = -0.0092, \quad \alpha_2 = 0.0080. \quad (94)$$

The same values of α_1 and α_2 can also be found by comparing T_0^0 , T_0^2 , and T_1^1 of ref.[30] with the corresponding combinations of eqs.(83,84). The numerical calculation shows that

the contribution of ρ -exchange to α_1 and α_2 is higher than the one from direct coupling by two order of magnitude. The contribution of ρ -exchange to T_2^0 and T_2^2 can be obtained from eqs.(84)

$$T_2^0 = \frac{4}{15\pi g^2 m_\rho^4} k^4, \quad T_2^2 = -\frac{2}{15\pi g^2 m_\rho^4} k^4.$$

Then we have

$$-\alpha_1 = \alpha_2 = \frac{1}{4} \frac{1}{g^2} \frac{f_\pi^4}{m_\rho^4} = 0.007.$$

Using the decay width of ρ meson(85), we obtain

$$\alpha_1 = -\alpha_2 = -3\pi \frac{f_\pi^4}{m_\rho^5} \frac{\Gamma_\rho}{\left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{\frac{2}{3}}}.$$

This is just the expression presented in ref.[8]. In present theory the reason of ρ dominance is the consequence of the cancellation between the original pion and the pion obtained from the shift of $a_\mu \rightarrow a_\mu - c\partial_\mu\pi$. On the other hand, the energy dependence of the amplitudes(83,84) have been predicted by present theory. The amplitudes $A(t, s, u)_\rho$ and $A(u, t, s)_\rho$ (84) predict the resonance structure of $\rho(770)$ in the channel of $I = 1$ and $l = 1$. The experimental data[32] clearly shows the $\rho(770)$ resonance in the amplitude T_1^1 . The comparison between theoretical predictions and experimental data are shown in Fig.1. From Fig.1, it can be seen that $|T_1^1|$ is in good agreement with data and T_2^0 , ReT_2^2 , and ReT_0^2 agree with data well. However, this theory does not provide an imaginary part for T_0^0 and the experimental data shows that there is an imaginary part in T_0^0 . On the other hand, the data shows(Table I)

that the theoretical predictions of a_0^0 and b_0^0 are lower than the experimental data. Therefore, in the channel of $I = 0$ and $l = 0$ something is missing in this theory. In ref.[33] the 0^{++} $f_0(1300)$ meson has been introduced to the effective meson theory to improve the theoretical value of a_0^0 . The $f_0(1300)$ meson can be introduced to the lagrangian(1), however, this is beyond the scope of this paper.

$\omega \rightarrow \rho\pi$ and other related processes

In 60's, Gell-Mann, Sharp and Wagner[34] used the coupling of $\omega \rightarrow \rho\pi$ to compute the decay rates of $\omega \rightarrow \pi\gamma$ and $\pi^0 \rightarrow \gamma\gamma$ in terms of VMD. The Syracuse group[12] has used the generalized Wess-Zumino action to study these processes.

In the process $\omega \rightarrow \rho\pi$ only pion is associated with γ_5 in this theory. Therefore, this process should be described by the effective lagrangian $\mathcal{L}_{IM}(6)$. In ref.[20] we use the lagrangian without vector and axial-vector mesons to calculate the effective baryon current $\frac{1}{3} < \bar{\psi}\gamma_\mu\psi >$ and it has been found that the current found in ref.[20] is just the topological current induced from the Wess-Zumino term. In this theory the interactions of ω meson with others are through the term $\frac{1}{g}\omega^\mu < \bar{\psi}\gamma_\mu\psi >$ in the lagrangian(1). It is the same with ref.[20] that the current $< \bar{\psi}\gamma_\mu\psi >$ can be bosonized by following equation

$$< \bar{\psi}\gamma_\mu\psi > = \frac{-i}{(2\pi)^D} \int d^D p Tr \gamma_\mu s_F(x, p). \quad (95)$$

The leading terms come from $n = 3$. After a lengthy and very careful derivation it has

been found that except the term $g\partial^2\omega_\mu$ in eq.(45) all other terms with even number of γ_5 in eq.(95) are cancelled each other. This is consistent with the fact that in two flavor case ω field is a flavor singlet except the kinetic term, ω field does not appear in $\mathcal{L}_{RE}(13)$ which describes the processes with normal parity. Using eq.(42) and taking $n = 3$, we have

$$\langle \bar{\psi}\gamma_\mu\psi \rangle = \frac{i}{(2\pi)^D} \int \frac{d^D p}{(p^2 - m^2)^4} \text{Tr} \gamma_\mu (\not{p} - m\hat{u}) \not{D}(\not{p} - m\hat{u}) \not{D}(\not{p} - m\hat{u}) \not{D}(\not{p} - m\hat{u}), \quad (96)$$

where $\not{D} = i\not{\partial} + \frac{1}{g}\not{p} + \frac{1}{g}(1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}}\not{\epsilon}\gamma_5$. In the calculation of eq.(96) we use dimension regularization. However, there is one term in eq.(96)

$$\frac{i}{(2\pi)^D} \int \frac{d^D p}{(2\pi)^D} \text{Tr} \gamma_\mu \not{p} \not{D} \not{p} \not{D} \not{p} \not{D} \not{p} \quad (97)$$

which has divergence and contains γ_5 . Therefore this part of the integral needs special treatment in dimension regularization. We use t'Hooft and Veltman's prescription[35] used to treat the triangle anomaly to calculate this integral. We define

$$p = \underline{p} + q$$

where \underline{p} has only four components in the 4-dimensional space and q is defined in $D - 4$ dimensional space. According to ref.[35] \underline{p} and q observe following equations

$$\underline{p}\gamma_5 = -\gamma_5\underline{p}, \quad \not{q}\gamma_5 = \gamma_5\not{q}.$$

Following these treatments the integral has been computed. The final expression of

$\langle \bar{\psi} \gamma_\mu \psi \rangle$ is

$$\begin{aligned}
\frac{1}{g} \omega^\mu \langle \bar{\psi} \gamma_\mu \psi \rangle = & g \partial^2 \omega \\
& + \frac{N_c}{(4\pi)^2 g} \varepsilon^{\mu\nu\alpha\beta} \omega_\mu \text{Tr} \left\{ \frac{4}{3g^2} [-v_{\nu\alpha} a_\beta + \partial_\nu (a_\alpha v_\beta) - i a_\nu a_\alpha a_\beta - \frac{i}{2} a_\alpha [\rho_\beta, \rho_\nu]] \right. \\
& - \frac{2}{3} (V_{\nu\alpha} a_\beta - a_{\nu\alpha} V_\beta) - \frac{1}{3} (V_{\nu\alpha} a_\beta + a_{\nu\alpha} V_\beta) \} \\
& + \frac{N_c}{(4\pi)^2 g} \frac{i}{3} \varepsilon^{\mu\nu\alpha\beta} \omega_\mu \text{Tr} \left\{ 2(D_\nu U) U^\dagger (D_\alpha U) U^\dagger (D_\beta U) U^\dagger - \frac{3i}{g^2} V_{\nu\alpha} a_\beta - \frac{3i}{g^2} a_{\nu\alpha} \rho_\beta \right. \\
& \left. + \frac{3i}{g} V_{\nu\alpha} [U(D_\beta U^\dagger) - U^\dagger(D_\beta U)] - \frac{3i}{g} a_{\nu\alpha} [U(D_\beta U^\dagger) + U^\dagger(D_\beta U)] \right\} \quad (98)
\end{aligned}$$

where $v_{\nu\alpha} = \partial_\nu \rho_\alpha - \partial_\alpha \rho_\nu - \frac{i}{g} [\rho_\nu, \rho_\alpha]$, $V_{\nu\alpha} = \partial_\nu \rho_\alpha - \partial_\alpha \rho_\nu - \frac{i}{g} [\rho_\nu, \rho_\alpha] - \frac{i}{g} [a_\nu, a_\alpha]$, and $a_\nu \rightarrow (1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}} a_\nu - c \partial_\nu \pi$. This expression can be simplified as

$$\begin{aligned}
\frac{1}{g} \omega^\mu \langle \bar{\psi} \gamma_\mu \psi \rangle = & \frac{N_c}{(4\pi)^2 g} \frac{2}{3} \varepsilon^{\mu\nu\alpha\beta} \omega_\mu \text{Tr} \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U U^\dagger \\
& + \frac{N_c}{(4\pi)^2 g} \frac{2}{3} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \text{Tr} \left\{ \frac{i}{g} [\partial_\beta U U^\dagger (\rho_\alpha + a_\alpha) - \partial_\beta U^\dagger U (\rho_\alpha - a_\alpha)] \right. \\
& \left. - \frac{2}{g^2} (\rho_\alpha + a_\alpha) U (\rho_\beta - a_\beta) U^\dagger - \frac{2}{g^2} \rho_\alpha a_\beta \right\}. \quad (99)
\end{aligned}$$

This formula is exact the same with the one obtained by Syracuse group[12]. In ref.[12] the Bardeen form of the anomaly[36] has been accepted and an arbitrary constant in their formula has been chosen to be 1. The authors of ref.[12] claim that their Wess-Zumino lagrangian with spin-1 fields agrees with Witten's[11] expression except an inadvertently omitted term. In eq.(99) all the couplings are fixed by g and c . This is the universality of coupling in this theory.

The interaction lagrangian of $\omega\rho\pi$ has been found from eq.(99)

$$\mathcal{L}_{\omega\rho\pi} = -\frac{N_c}{\pi^2 g^2 f_\pi} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \rho_\alpha^i \partial_\beta \pi^i, \quad (100)$$

which can be used to study related processes. Using the vertex of $\omega \rightarrow \pi\gamma$ obtained by the substitution $\rho_\mu^0 \rightarrow \frac{e}{f_\rho} A_\mu$ in eq.(100) the decay width has been calculated to be

$$\Gamma(\omega \rightarrow \gamma\pi) = 724 keV.$$

The experimental value is $717(1 \pm 0.07) keV$. In the same way, we obtain the vertex of $\rho \rightarrow \pi\gamma$ by the substitution $\omega_\mu \rightarrow \frac{e}{f_\omega} A_\mu$ in eq.(100). The decay width calculated is

$$\Gamma(\rho^0 \rightarrow \pi^0 \gamma) = 76.2 keV.$$

The experimental data is $68.2(1 \pm 0.12) keV$.

$\pi^0 \rightarrow \gamma\gamma$ is an evidence of the Adler-Bell-Jackiw anomaly[37]. This process is a crucial test of present theory. According to VMD, the $\pi^0\gamma\gamma$ vertex should be obtained by using the substitutions(32) in eq.(100).

$$\mathcal{L}_{\pi^0 \rightarrow \gamma\gamma} = -\frac{\alpha}{\pi f_\pi} \varepsilon^{\mu\nu\alpha\beta} \pi^0 \partial_\mu A_\nu \partial_\alpha A_\beta, \quad (101)$$

which is the expression given by Adler-Bell-Jackiw anomaly[37]. The decay width obtained from eq.(101) is

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{\alpha^2}{16\pi^3} \frac{m_\pi^3}{f_\pi^2}, \quad (102)$$

which is the result of triangle anomaly. The numerical result is 7.64eV and the data is 7.74(1 ± 0.072)eV. If one of the two photons $\pi^0 \rightarrow \gamma\gamma$ is virtual, according to VMD, there are vector meson poles in the decay amplitude. The amplitude has been written as

$$\mathcal{M} = \frac{2\alpha}{\pi f_\pi} \left\{ 1 + \frac{1}{2} \frac{q^2}{m_\rho^2 - q^2} + \frac{1}{2} \frac{q^2}{m_\omega^2 - q^2} \right\}, \quad (103)$$

where q is the momentum of the virtual photon. Due to q^2 is much less than the mass of the vector meson, the form factor takes following form approximately

$$\begin{aligned} F(q^2) &= 1 + \frac{1}{2} \frac{q^2}{m_\rho^2 - q^2} + \frac{1}{2} \frac{q^2}{m_\omega^2 - q^2} = 1 + a \frac{q^2}{m_{\pi^0}^2}, \\ a &= \frac{m_{\pi^0}^2}{2} \left(\frac{1}{m_\omega^2} + \frac{1}{m_\rho^2} \right) = 0.03. \end{aligned} \quad (104)$$

The data is $a = 0.032 \pm 0.004$.

In $\omega \rightarrow \pi\pi\pi$ besides the process $\omega\rho\pi$ and $\rho \rightarrow \pi\pi$ there is direct coupling $\omega\pi\pi\pi$ which has been derived from eq.(99)

$$\mathcal{L}_{\omega\pi\pi\pi} = \frac{2}{g\pi^2 f_\pi^3} \left(1 + \frac{6c^2}{g^2} - \frac{6c}{g} \right) \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{ijk} \omega_\mu \partial_\nu \pi_i \partial_\alpha \pi_j \partial_\beta \pi_k. \quad (105)$$

In both the $\mathcal{L}_{\omega\pi\pi\pi}$ and $\mathcal{L}_{\omega \rightarrow \rho\pi}$ there are a factor of $\frac{1}{\pi^2}$. Therefore, qualitatively speaking, this theory predicts a narrower width for ω decay. Using the formula of c(25) and the value of g it is found that

$$1 + \frac{6c^2}{g^2} - \frac{6c}{g} = -0.083.$$

The last two terms come from the shift $a_\mu \rightarrow a_\mu - c\partial_\mu\pi$ and there is very strong cancellation.

Using eqs.(49,100,105) the decay width has been obtained

$$\Gamma(\omega \rightarrow \pi\pi\pi) = \frac{1}{24m_\omega(2\pi)^3} \int dq_1^2 dq_2^2 \{ |\vec{p}_1|^2 |\vec{p}_2|^2 - (\vec{p}_1 \cdot \vec{p}_2)^2 \} \\ \{ 3f_{\omega 3\pi} + f_{\omega\rho\pi} f_{\rho\pi\pi} (\frac{1}{q_1^2 - m_\rho^2} + \frac{1}{q_2^2 - m_\rho^2} + \frac{1}{q_3^2 - m_\rho^2}) \}, \quad (106)$$

where

$$f_{\omega 3\pi} = \frac{2}{g\pi^2 f_\pi^3} (1 + \frac{6c^2}{g^2} - \frac{6c}{g}), \quad f_{\omega\rho\pi} = \frac{N_c}{\pi^2 g^2 f_\pi}, \quad (107)$$

and $q_i^2 = (p - p_i)^2$, p is ω momentum and p_i is pion momentum. We obtain

$$\Gamma(\omega \rightarrow 3\pi) = 5MeV.$$

If only $\omega \rightarrow \rho\pi$ and $\rho \rightarrow \pi\pi$ are taken into account the width of $\omega \rightarrow 3\pi$ is 5.4MeV. The experimental value is $7.43(1 \pm 0.02)MeV$. From this study we can see that the process of $\omega \rightarrow \rho\pi$ is dominant the decay of $\omega \rightarrow 3\pi$ as proposed by the authors[34]. The direct coupling of $\omega \rightarrow 3\pi$ is responsible for about 20% of the decay rate. The agreement between theoretical and experimental decay rates of $\pi^0 \rightarrow \gamma\gamma$, $\omega \rightarrow \pi\gamma$, and $\rho \rightarrow \pi\gamma$ shows that $\omega \rightarrow \rho\pi$ obtained in this theory is more reliable. However, at tree level due to the cancellation $f_{\omega 3\pi}$ is too small and has a wrong sign. Therefore, corrections from loop diagrams and terms with higher order derivatives to $f_{\omega 3\pi}$ are needed.

The decays of $f_1(1285)$ meson

In this theory $f_1(1285)$ meson is the chiral partner of ω meson(see eq.(1)). The decay of

$f_1 \rightarrow 4\pi$ consists of two processes: direct coupling $f_1 \rightarrow \rho\pi\pi$ and $f_1 \rightarrow a_1\pi \rightarrow \rho\pi\pi$. f_1 meson is associated with γ_5 (see eq.(1)), hence, odd number of γ_5 are involved in these two processes. Therefore, the decay of $f_1 \rightarrow \rho\pi\pi$ can not be found in eq.(13) and should be found in the Wess-Zumino lagrangian with vector and axial-vector mesons. In this paper the method used to find the effective lagrangian of these processes is the same with the one used to study the decays of ω meson. From the lagrangian(1) and normalization of f_1 meson the vertices of f_1 decays can be found from following formula

$$\mathcal{L}_i = \frac{1}{g} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} f_\mu < \bar{\psi} \gamma_\mu \gamma_5 \psi > . \quad (108)$$

The bosonization of $< \bar{\psi} \gamma_\mu \gamma_5 \psi >$ can be carried out by using following equation

$$< \bar{\psi} \gamma_\mu \gamma_5 \psi > = \frac{-i}{(2\pi)^D} \int d^D p \text{Tr} \gamma_\mu \gamma_5 s_F(x, p). \quad (109)$$

Substituting eq.(42) into eq.(109) the flavor singlet axial-vector current of mesons can be achieved. Due to the fact that only odd number of γ_5 is involved in $f_1 \rightarrow \rho\pi\pi$, we are only interested in the terms with the antisymmetric tensor. The leading terms with antisymmetric tensor appear at $n = 3$. It is similar to the case of ω meson there are divergent terms which can be treated by the prescription provided in ref.[35]. Finally, the terms with $\varepsilon^{\mu\nu\alpha\beta}$ in effective lagrangian(108) take following form

$$\mathcal{L}_i = -\frac{N_c}{3g^3(4\pi)^2} \left(1 - \frac{1}{2\pi^2 g^3}\right)^{-\frac{1}{2}} \varepsilon^{\mu\nu\alpha\beta} f_\mu \text{Tr}(3V_{\nu\alpha} V_\beta - a_{\nu\alpha} a_\beta)$$

$$\begin{aligned}
& + \frac{iN_c}{3g^2(4\pi)^2} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \varepsilon^{\mu\nu\alpha\beta} f_\mu \text{Tr} \{ -\partial_\alpha (\partial_\nu U U^\dagger d_\beta^+ + \partial_\nu U^\dagger U d_\beta^-) \\
& - 2(\partial_\nu U U^\dagger \partial_\alpha U U^\dagger d_\beta^+ + \partial_\nu U^\dagger U \partial_\alpha U^\dagger U d_\beta^-) \\
& - \frac{i}{g} (d_\nu^+ \partial_\alpha d_\beta^+ + d_\nu^- \partial_\alpha d_\beta^- + 2U^\dagger d_\nu^+ U \partial_\alpha d_\beta^- + 2U d_\nu^- U^\dagger \partial_\alpha d_\beta^+ - 2\partial_\nu U U^\dagger d_\alpha^+ d_\beta^+ - 2\partial_\nu U^\dagger U d_\alpha^- d_\beta^-) \\
& - \frac{1}{g^2} (2U d_\nu^- U^\dagger d_\alpha^+ d_\beta^+ + 2U^\dagger d_\nu^+ U d_\alpha^- d_\beta^- + d_\nu^- d_\alpha^- d_\beta^- + d_\nu^+ d_\alpha^+ d_\beta^+) \}. \tag{110}
\end{aligned}$$

From this lagrangian, the same conclusion with ref.[14] has been reached that the decays of $f_1 \rightarrow \rho\rho$ and $\omega\omega$ are forbidden. Therefore, f_1 meson can not decay to two real photons. This is Yang's theorem[38].

The lagrangians of the decay of $f \rightarrow a_1\pi$ and $f \rightarrow \rho\pi\pi$ have been found from eq.(110)

$$\begin{aligned}
\mathcal{L}_{fa_1\pi} &= -\frac{4N_c}{3(4\pi)^2 f_\pi g^2} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1} \varepsilon^{\mu\nu\alpha\beta} f_\mu \partial_\nu \pi^i \partial_\alpha a_\beta^i \\
\mathcal{L}_{f\rho\pi\pi} &= \frac{4N_c}{3(4\pi)^2 g^2 f_\pi^2} \left(1 - \frac{4c}{g}\right) \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \varepsilon^{\mu\nu\alpha\beta} \epsilon_{ijk} \\
&\quad \{ \partial_\alpha (\partial_\nu \pi_i \pi_j \rho_\beta^k) - 2\partial_\nu \pi_i \partial_\alpha \pi_j \rho_\beta^k \}. \tag{111}
\end{aligned}$$

In both $\mathcal{L}_{f\rho\pi\pi}$ and $\mathcal{L}_{fa_1\pi}$ there are a factor of $\frac{1}{\pi^2}$. Therefore, this theory predicts a narrower width for $f \rightarrow \rho\pi\pi$. On the other hand, the factor of $1 - \frac{4c}{g}$ in $\mathcal{L}_{f\rho\pi\pi}$ is very small, hence the process $f \rightarrow a_1\pi \rightarrow \rho\pi\pi$ is dominant the decay of $f \rightarrow \rho\pi\pi$. Using eqs.(61,111) the decay width has been calculated

$$\Gamma(f_1 \rightarrow \rho\pi\pi) = 6.01 \text{ MeV}. \tag{112}$$

The experimental value of the decay width is $6.96(1 \pm 0.33) \text{ MeV}$. The prediction of $\Gamma(f_1 \rightarrow$

$\rho\pi\pi$) is in agreement with data.

In another decay of $f_1(1285)$ meson, $f \rightarrow \eta\pi\pi$ (excluding $a_0(980)\pi$), even number of γ_5 are involved. Therefore, the effective lagragian of this decay should be found from the lagrangian $\mathcal{L}_{RE}(13)$. Of course, the vertex of $f\eta\pi\pi$ can also be found from eq.(108) and it should be the same with the one obtained from eq.(13). The calculation shows that in the lagrangian \mathcal{L}_{RE} the term at the second order in derivatives

$$\frac{F^2}{16} Tr D_\mu U D^\mu U^\dagger$$

does not contribute to this decay. The effective lagrangian of this decay comes from the terms at the fourth order in derivatives which have a factor of $\frac{1}{(4\pi)^2}$. Therefore, this theory predicts a narrow width for the process $f_1 \rightarrow \eta\pi\pi$. In the chiral limit, the effective lagrangian has been found from eq.(13)

$$\mathcal{L}_{f\eta\pi\pi} = \frac{4N_c}{3(4\pi)^2 f_\pi^3} \frac{1}{g} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-\frac{1}{2}} \left(1 - \frac{2c}{g}\right)^3 0.7104 f_\mu \{ \partial^\mu \eta \partial_\nu \pi_i \partial^\nu \pi_i + 2 \partial_\nu \eta \partial^\nu \pi_i \partial^\mu \pi_i \}, \quad (113)$$

where the factor 0.7104 is from the mixing between η and η' $0.7104 = \frac{1}{\sqrt{3}}(\cos\theta - \sqrt{2}\sin\theta)$ and $\theta = -10^\circ$. The numerical result of the decay width is

$$\Gamma(f \rightarrow \eta\pi\pi) = 27.5 keV. \quad (114)$$

Theoretical prediction of the branch ratio is $1.15 \times 10^{-3}(1 \pm 0.13)$ and the data of the branch ratio is $(10_{-6}^{+7})\%$. In principle, the meson $a_0(980)(1^-(0^{++}))$ can be incorporated into the

lagrangian, then the decay of $f \rightarrow a_0\pi$ can be studied. However, this is beyond the scope of present paper.

Using VMD and $\mathcal{L}_{f\rho\pi\pi}$, the decay width of $f \rightarrow \gamma\pi\pi$ has been calculated

$$\Gamma(f \rightarrow \gamma\pi\pi) = 18.5keV. \quad (115)$$

In this theory the decay of $f_1 \rightarrow virtualphoton + \rho$ involves loop diagrams whose calculation is beyond the scope of this paper.

The decays of $\rho \rightarrow \eta\gamma$ and $\omega \rightarrow \eta\gamma$

According to VMD, the decays of $\rho \rightarrow \eta\gamma$ and $\omega \rightarrow \eta\gamma$ are related to the vertices $\eta\rho\rho$ and $\eta\omega\omega$ in which odd number of γ_5 are involved and these vertices can not be found from \mathcal{L}_{RE} (eq.(13)). From the lagrangian(1) it can be seen that the interaction between η and other mesons can be found from

$$\begin{aligned} \mathcal{L}_\eta &= -\frac{2i}{f_\pi} 0.7104m\eta <\bar{\psi}\gamma_5\psi>, \\ <\bar{\psi}\gamma_5\psi> &= \frac{-i}{(2\pi)^D} \int d^Dp Tr\gamma_5 s_F(x,p). \end{aligned} \quad (116)$$

Substituting the solution(42) into the equation(116) \mathcal{L}_η has been obtained. The leading terms come from $n = 4$. We are only interested in the terms containing the vertices ηvv , which have been found

$$<\bar{\psi}\gamma_5\psi> = \frac{N_c}{(4\pi)^2} \frac{i}{6mg} \varepsilon^{\mu\nu\alpha\beta} Tr(F_{\mu\nu}^+ D_\alpha^- D_\beta^+ + F_{\mu\nu}^- D_\alpha^+ D_\beta^-)$$

$$\begin{aligned}
& + \frac{N_c}{(4\pi)^2} \frac{i}{6mg} \varepsilon^{\mu\nu\alpha\beta} \text{Tr}(D_\mu^+ D_\nu^- F_{\alpha\beta}^+ + D_\mu^- D_\nu^+ F_{\alpha\beta}^-) \\
& + \frac{N_c}{(4\pi)^2} \frac{i}{3m} \varepsilon^{\mu\nu\alpha\beta} \text{Tr}(D_\mu^+ D_\nu^- D_\alpha^+ D_\beta^- + D_\mu^- D_\nu^+ D_\alpha^- D_\beta^+), \\
& D_\mu^\pm = \partial_\mu - \frac{i}{g} d_\mu^\pm, \quad d_\mu^\pm = v_\mu \mp a_\mu, \quad a_\mu \rightarrow (1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}} a_\mu - c \partial_\mu \pi, \\
& F_{\mu\nu}^\pm = \partial_\mu d_\nu^\pm - \partial_\nu d_\mu^\pm - \frac{i}{g} [d_\mu^\pm, d_\nu^\pm].
\end{aligned} \tag{117}$$

The vertices of $\eta\rho\rho$ and $\eta\omega\omega$ are revealed from eqs.(117)

$$\mathcal{L}_{\eta vv} = -\frac{8}{g^2 f_\pi} \frac{N_c}{(4\pi)^2} \varepsilon^{\mu\nu\alpha\beta} 0.7104 \eta (\partial_\mu \rho_\nu^i \partial_\alpha \rho_\beta^i + \partial_\mu \omega_\nu \partial_\alpha \omega_\beta). \tag{118}$$

Using VMD we obtain

$$\begin{aligned}
\mathcal{L}_{\rho\eta\gamma} &= -\frac{8e}{gf_\pi} \frac{N_c}{(4\pi)^2} \varepsilon^{\mu\nu\alpha\beta} 0.7104 \eta \partial_\mu \rho_\nu^0 \partial_\alpha A_\beta, \\
\mathcal{L}_{\omega\eta\gamma} &= -\frac{8e}{3gf_\pi} \frac{N_c}{(4\pi)^2} \varepsilon^{\mu\nu\alpha\beta} 0.7104 \eta \partial_\mu \omega_\nu \partial_\alpha A_\beta.
\end{aligned} \tag{119}$$

The decay widths of $\rho \rightarrow \eta\gamma$ and $\omega \rightarrow \eta\gamma$ have been calculated

$$\begin{aligned}
\Gamma(\rho \rightarrow \eta\gamma) &= 46.1 \text{keV}, \quad B(\rho \rightarrow \eta\gamma) = 3.04 \times 10^{-4}, \\
\Gamma(\omega \rightarrow \eta\gamma) &= 5.87 \text{keV}, \quad B(\omega \rightarrow \eta\gamma) = 6.96 \times 10^{-4}.
\end{aligned} \tag{120}$$

The experimental data are

$$B(\rho \rightarrow \eta\gamma) = (3.8 \pm 0.7) \times 10^{-4}, \quad B(\omega \rightarrow \eta\gamma) = (8.3 \pm 2.1) \times 10^{-4}, \tag{121}$$

Theoretical predictions are in good agreement with data. For $\eta \rightarrow \gamma\gamma$ besides $\eta \rightarrow \rho\rho$ and

$\eta \rightarrow \omega\omega$, the process $\eta \rightarrow \phi\phi$ also contributes to $\eta \rightarrow \gamma\gamma$, we will study $\eta \rightarrow \gamma\gamma$ in another paper in which the strange flavor is included.

After the study of those physical processes, three problems of this theory should be discussed. They are: loop diagrams, dynamical chiral symmetry breaking, and momentum expansion.

Large N_c expansion

According to t'Hooft[6], in large N_c limit QCD is equivalent to a meson theory at low energy. Therefore, large N_c expansion plays a crucial role in the connection between QCD and effective meson theory, even though we do not know how to derive the lagrangian of effective meson theory from QCD directly. In present theory the large N_c expansion plays an important role too. The quark fields in lagrangian(1) carry colors. In order to obtain the effective lagrangian of mesons from the lagrangian(1), the quark fields have been integrated out by path integral. After this integration the trace in the color space generates the number of color N_c . Parameter m of the lagrangian(1) is $O(1)$ in large N_c expansion. Eq.(14) determines that F^2 is order of N_c , hence f_π is order of $O(\sqrt{N_c})$. The coupling constant g defined by eq.(15) is $O(\sqrt{N_c})$. After normalization, the physical meson fields, pion, η , ρ , ω , a_1 , and f_1 are all at order of $O(\sqrt{N_c})$. It is needed to point out that the original form of the factor $(1 - \frac{1}{2\pi^2 g^2})^{-\frac{1}{2}}$ in the normalization of axial-vector meson is $(1 - \frac{N_c}{6\pi^2 g^2})^{-\frac{1}{2}}$. Therefore, this factor is at order of $O(1)$. The masses of mesons are at order of $O(1)$. Using all these

results, it is not difficult to find out that all the vertices of this paper are at order of N_c and it is obvious that the propagator of meson is order of $O(1)$. Therefore, order of magnitude of a Feynman diagram of mesons in large N_c expansion is given by

$$N_c^{N_v - N_p}, \quad (122)$$

where N_v is the number of vertices and N_p is the number of internal lines. Eq.(122) tells that all tree diagrams are at order of $O(N_c)$, hence they are leading contributions. A diagram with loops is at higher order in large N_c expansion. For instance, a diagram of one loop with two internal lines is order of $O(1)$. In this paper all calculations have been done at tree level. Most of the theoretical predictions are in agreement with data. This success can be viewed as a support of the large N_c expansion. However, in some cases due to cancellation, for instance the direct coupling of $\omega \rightarrow 3\pi$, or other reasons the leading term is very small a correction of loop diagram should be taken.

Dynamical chiral symmetry breaking

The parameter m in eq.(1) is associated with quark condensate which is defined as

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = -\frac{i}{(2\pi)^D} \int d^D p \text{Tr} \langle 0 | s_F(p, x) | 0 \rangle. \quad (123)$$

At tree level, using eq.(42) we obtain the relation between m and quark condensate

$$\langle 0 | \bar{\psi}(x) \psi(x) | 0 \rangle = 3m^3 g^2 \left(1 + \frac{1}{2\pi^2 g^2}\right). \quad (124)$$

This is u and d quark condensate. Nonzero quark condensate means dynamical chiral symmetry breaking. Therefore, there is dynamical chiral symmetry breaking in this theory. On the other hand, the quark mass term $-\bar{\psi}M\psi$ can be introduced to the lagrangian(1), where M is the mass matrix of u and d quark. Using eqs.(40,42) and removing a constant away the leading term in quark mass expansion has been obtained

$$- \langle \bar{\psi}M\psi \rangle = \frac{i}{(2\pi)^D} \int d^D p \text{Tr} M s_F(x, x) = -\frac{1}{f_\pi^2} (m_u + m_d) \langle 0 | \bar{\psi}\psi | 0 \rangle . \quad (125)$$

The pion mass(79) revealed from this equation is

$$m_\pi^2 = -\frac{2}{f_\pi^2} (m_u + m_d) \langle 0 | \bar{\psi}\psi | 0 \rangle . \quad (126)$$

Detailed discussion of masses of pseudoscalar mesons can be found in ref.[31]. Eq.(126) is a well known formula obtained by the theory of chiral symmetry breaking proposed by Gell-Mann, Oakes, and Renner[39], and by Glashow and Weinberg[40].Eq.(126) tells that the quark condensate is negative, hence the parameter m is negative too. The parameter m has been determined from eqs.(15,25,26)

$$m = -300 \text{MeV}, \quad (127)$$

and eq.(124) determines

$$\langle 0 | \bar{\psi}\psi | 0 \rangle = -(241 \text{MeV})^3. \quad (128)$$

The quark mass is determined to be

$$m_u + m_d = 23.5 MeV.$$

It is about twice the current value[41]. As pointed in ref.[41], the determination of absolute value of quark mass is model dependent.

Derivative expansion

This theory is an effective meson theory at low energies. Like the chiral perturbation theory[1], the derivative expansion (to be accurate, covariant derivative expansion) has been applied. It seems that the derivative expansion works in the studies presented in this paper. The calculations of decay widths are good examples. If the terms at the second order in derivatives in lagrangian(13) contribute to the decay of a meson, the decay width of this meson is broader. ρ decay and a_1 decay are two examples. If only the terms at the fourth order in derivatives contribute to the decay, the decay width is narrower. The reason is that in the decay amplitude there is a factor of $\frac{1}{\pi^2}$. The predictions of narrower widths for decays, $\omega \rightarrow 3\pi$, $f \rightarrow \rho\pi\pi$, and $f \rightarrow \eta\pi\pi$, support this argument. From table I it can be seen that the scattering lengths and slopes(a_0^0 , b_0^0 , a_1^1 , a_0^2 , b_0^2), which are obtained from the terms at the zeroth order or the second order in derivatives, are much greater than a_2^0 , a_2^2 , and b_1^1 which are from the terms at the fourth order in derivatives.

This theory is an effective theory and it is not renormalizable, as mentioned above, a cut-

off of momentum has to be introduced into this theory. The eq.(15) can be used to determine the cut-off. Using cut-off instead dimension regularization, eq.(15) has been rewritten as

$$\frac{N_c}{(4\pi)^2} \left\{ \log\left(1 + \frac{\Lambda^2}{m^2}\right) + \frac{1}{1 + \frac{\Lambda^2}{m^2}} - 1 \right\} = \frac{1}{16} \frac{F^2}{m^2} = \frac{3}{8} g^2. \quad (129)$$

Using the values of m and g , we obtain

$$\Lambda = 1.6 GeV. \quad (130)$$

Derivative expansion is momentum expansion and Λ is the maximum momentum. The momentum expansion requires that the momentum is less than Λ . The masses of ρ , ω , a_1 , and f_1 are less than Λ . However, there is physical case in which momentum expansion is not suitable. In $\pi\pi$ scattering there is ρ resonance in the scattering amplitudes(84). At very low energy the momentum expansion has been applied to $\pi\pi$ scattering amplitudes, however, in the region of ρ resonance the momentum expansion is not working because it destroys the resonance. Therefore, we do not apply the momentum expansion to the amplitudes of resonance in this paper.

Summary of the results

To summarize the results achieved by this theory. VMD has been revealed from this theory. Weinberg's first sum rule and new relations about a_1 meson decay are satisfied. KSFR sum rule is satisfied pretty well. New mass relations between vector and axial-vector mesons have been found. Weinberg's $\pi\pi$ scattering lengths and slopes have been revealed. The amplitude

of $\pi^0 \rightarrow \gamma\gamma$ obtained by this theory is exact the same with the prediction of triangle anomaly. This theory provides a united description of the physical processes with normal and abnormal parity and the universality of coupling has been revealed. The effective lagrangians used to treat the processes of abnormal parity are exactly the same with the ones obtained from gauging Wess-Zumino lagrangian with vector and axial-vector mesons. In chiral limit, we take f_π and m_ρ as inputs. The coupling constant g is chosen to fit the data. In phase spaces physical m_π and m_η (m_π and m_η are inputs) have been taken and in the amplitudes only the leading terms have been kept in chiral perturbation. The results of $\pi\pi$ scattering have been shown in Table I and Fig.1. Other results are listed in Table II.

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Table 2: Table II Summary of the results

	Experimental	Theoretical
f_π	0.186GeV	input
m_ρ	$769.9 \pm 0.8 \text{ MeV}$	input
m_π	0.138 Gev	input
m_η	$547.45 \pm 0.19 \text{ MeV}$	input
g		0.35 input
m_ω	$781.94 \pm 0.12 \text{ MeV}$	0.77 GeV
m_a	$1230 \pm 40 \text{ MeV}$	1.389 GeV
m_{f_1}	$1282 \pm 5 \text{ MeV}$	1.389 GeV
π form factor	consistent with ρ pole	ρ pole
radius of π	$0.663 \pm 0.023 \text{ fm}$	0.63 fm
$g_{\rho\gamma}$	$0.116(1 \pm 0.05) \text{ GeV}^2$	0.104 GeV^2
$g_{\omega\gamma}$	$0.0359(1 \pm 0.03) \text{ GeV}^2$	0.0357 GeV^2
$\Gamma(\rho \rightarrow \pi\pi)$	$151.2 \pm 1.2 \text{ MeV}$	135. MeV
$\Gamma(\omega \rightarrow \pi\pi)$	$0.186(1 \pm 0.15) \text{ MeV}$	0.136MeV

$\Gamma(a_1 \rightarrow \rho\pi)$	$\sim 400 \text{ MeV}$	325 MeV
$\Gamma(a_1 \rightarrow \gamma\pi)$	$(640 \pm 246) \text{ keV}$	252 keV
$\frac{d}{ds}(a_1 \rightarrow \rho\pi)$	-0.11 ± 0.02	-0.097
$\Gamma(\tau \rightarrow a_1\nu)$	$(2.42 \pm 0.76) \times 10^{-13} \text{ GeV}$	$1.56 \times 10^{-13} \text{ GeV}$
$\Gamma(\tau \rightarrow \rho\nu)$	$(0.495 \pm 0.023) \times 10^{-12} \text{ GeV}$	$4.84 \times 10^{-13} \text{ GeV}$
$\Gamma(\pi^0 \rightarrow \gamma\gamma)$	$7.74(1 \pm 0.072) \text{ eV}$	7.64 eV
a(form factor of $\pi^0 \rightarrow \gamma\gamma$)	0.032 ± 0.004	0.03
$\Gamma(\omega \rightarrow \pi\gamma)$	$717(1 \pm 0.07) \text{ keV}$	724 keV
$\Gamma(\rho \rightarrow \pi\gamma)$	$68.2(1 \pm 0.12) \text{ keV}$	76.2 keV
$\Gamma(\omega \rightarrow \pi\pi\pi)$	$7.43(1 \pm 0.02) \text{ MeV}$	5 MeV
$\Gamma(f_1 \rightarrow \rho\pi\pi)$	$6.96(1 \pm 0.33) \text{ MeV}$	6.01 MeV
$B(f_1 \rightarrow \eta\pi\pi)$	$(10^{+7}_{-6})\%$	1.15×10^{-3}
$\Gamma(f_1 \rightarrow \gamma\pi\pi)$		18.5 keV
$B(\rho \rightarrow \gamma\eta)$	$(3.8 \pm 0.7) \times 10^{-4}$	3.04×10^{-4}
$B(\omega \rightarrow \gamma\eta)$	$(8.3 \pm 2.1) \times 10^{-4}$	6.96×10^{-4}